TexMATYC News

Texas Mathematical Association of Two –Year Colleges
Affiliate to the American Mathematical Association of Two-Year Colleges

Fall 2007 www.texmatyc.org

Inside this issue:

President's Message

Mel Griffin, Walden University

There are many issues taking place in Texas public schools that will impact the work we do in community college mathematics. Two of the most important education bills passed by the 2007 Legislature relate to changing the high school graduation requirements and changing the accountability testing that has been in place for the last few years.

Students who enter the ninth grade during the 2007-2008 academic year are required to take four years of mathematics to earn a high school diploma under either the Recommended High School Program (RHP) or the Distinguished Achievement High School Program (DAP). Both of these graduation plans require a student to take Algebra I, Algebra II, Geometry, and one additional mathematics course. The state default curriculum is the RHP. However, if parents and the high school principal request that a student be exempt from the RHP, the student can graduate with the Minimum High School Program (MHP) that requires only three years of mathematics, which must include Algebra I, and Geometry.

A student following the DAP must take a fourth year of mathematics that has an Algebra II prerequisite. This may include Independent Study in Mathematics, Precalculus, Calculus, Statistics, International Baccalaureate advanced courses, AP Computer Science, or concurrent enrollment in college mathematics courses. In addition to these current choices, another course for the fourth year in mathematics is being developed that is specifically designed for those students who do not meet the college readiness standard.

A student who follows the RHP may take any of the courses listed for the DAP or a course called Mathematical Models with Applications for the fourth year of mathematics. If a student takes Math Models for the fourth course, it must be taken prior to Algebra II. Many high schools use the Math Models course to help students pass the exit level Texas Assessment of Knowledge and Skills (TAKS). Students selecting this option will take Algebra II as their last mathematics course.

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College Algebra may be used as the fourth year mathematics course if it completely covers the Texas Essential Knowledge and Skills (TEKS) for one of the courses that is allowable for a fourth year course. The obvious choice is matching College Algebra with the high school TEKS for Precalculus. Students would be awarded dual credit for Precalculus and College Algebra. The decision as to whether or not College Algebra can be used as a dual credit course for Precalculus is made by each individual school district in collaboration with the institution of higher education offering the course. The Texas Education Agency does not interject into these decisions. In addition to Precalculus, College Algebra may be used for a dual-credit course if the course covers the TEKS for Independent Study in Mathematics.

The other notable legislation that will affect community college mathematics is the change from the current grade-level (9-11) TAKS test to end-of-course exams. In mathematics these tests will be Algebra I, Geometry, and Algebra II. The freshman class of 2011-2012 will be the first students to take the new end-of-course exams. An Algebra I end-of-course exam has been available for several years and a released version can be found on the TEA website. The geometry end-of-course exam was field tested in the spring of 2007. The Algebra II test is still in the planning stages.

This legislation dictates that the end-of-course exam "shall be worth 15% of the student's final grade in the courses required for graduation". Students must have a cumulative score that is at least equal to the product of the number of tests taken in a subject and 70 in order to graduate. Each test will be scored between 0 – 100 points. Students must score at least 60 to count that test score in the cumulative average. Additionally, end-of-course exams will have separate test items that are intended to measure college readiness and the need for developmental courses.

Also included in the legislation is a provision that all students will take a college readiness diagnostic exam in the eighth and tenth grades. All students will take a college entrance test in the eleventh grade at state expense.

Although these changes may seem to be years away, we need to be aware of these transitions in the public schools and plan for the effects these changes will have in developmental education as well as college level mathematics courses. More detailed information can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us).

I hope to see all of you in Dallas for the TexMATYC sessions in conjunction with the Texas Community College Teachers Association (TCCTA) Conference in February 2008.

Mel Griffin

TexMATYC: What IF?

Paula Wilhite, Vice President

What if we increased our membership to 2000+, a ten-fold increase of our current membership? We can only imagine the strength of our voice with this magnitude of individuals who strongly believe in the importance of mathematics education. We can only dream of a community of educators who have professional development available for all - whether full-time or part-time, whether college-level or developmental education, and whether experienced or not. We can only hope during our lifetime to experience positive change towards a viable and meaningful program of study.

I chose the number 2000 by estimating the number of full-time / part-time two-year college mathematics educators across the state. How many math instructors at your college are members of TexMATYC? When your campus becomes 100%, your college will be recognized for this accomplishment.

For the year 2007 - 2008, TexMATYC is planning many activities to support your interests and needs in the classroom. Plans include online professional development, TCCTA / TexMATYC annual conference, regular newsletters, collaborative networking, and representation with updates in P-16 initiatives, AMATYC, and statewide studies in math education. At $5 per year, this is no doubt the bargain of bargains in professional organizations! It is the result of the dreams and hopes of individuals just like you who want to make a difference.

Please help us start the year with success!

Your Feedback is Requested

Advanced Quantitative Decision-Making
A New Fourth-Year Mathematics Course for Texas

Charles A. Dana Center at The University of Texas at Austin and
Texas Association of Supervisors of Mathematics
Submitted by Cathy Seeley

In recent years, many educators and others have recognized the need for an additional—and nonremedial—path through high school mathematics as an alternative to the precalculus/calculus path. With Texas moving to a four-year mathematics requirement for high school, this need has risen to a new level of urgency.

Schools are struggling to find options for all students, especially for those students who may not be headed toward a career in science, technology, engineering, or mathematics (STEM) fields. In designing options for all students, balancing rigor, relevance, and accessibility of mathematics content has become increasingly challenging and increasingly necessary.

The Charles A. Dana Center at The University of Texas at Austin, in cooperation with the Texas Association of Supervisors of Mathematics (TASM), has received a grant from the Greater Texas Foundation to develop a mathematics course that would follow Algebra II and Geometry and could fulfill this need.

During the 2007-2008 school year, a working group of about 25 educators representing high school and post-secondary mathematics will create a course description and student expectations for a course called Advanced Quantitative Decision-Making. The course is intended to be available for implementation in the 2009–2010 school year.

The working group will also develop a basic set of online instructional resources. Ideally, commercial publishers and others will create more comprehensive instructional materials as the course is being developed. The Dana Center and TASM will also seek funding for extensive professional development and teacher support for implementing the new course.

During this year of development, the Dana Center and TASM will seek your ideas on the fourth-year mathematics course during two intensive feedback periods. During the first period, we will coordinate focus groups and other outreach activities **September through November 2007**, **to collect feedback on an elaborated description of the new fourth-year mathematics course**.

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The Charles A. Dana Center
The University of Texas at Austin

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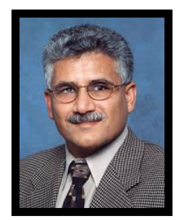
As the working group develops more specific student expectations in December, we will work from **January through April 2008 to seek feedback on the draft student expectations.** Ideally, these expectations will be in a form that could eventually become TEKS for the course if adopted by the State Board of Education.

This development year provides a great opportunity for local communities to convene prekindergarten–16 groups around identifying the mathematics that students need as they leave high school. In particular, this course can be a focus of discussion and collaboration between high school mathematics teachers and those who teach mathematics at the two- or four-year college level.

The Dana Center and TASM welcome your input and encourage you to participate in development of this course. Consider convening focus groups or hosting discussion sessions around this course. Throughout the 2007–2008 school year, you can keep up with how the development is progressing, access the current discussion draft, and submit your feedback by accessing this website: **utdanacenter.org/aqd**.

TexMATYC Financial Report (Ending 9/11/07)

Habib Far, Treasurer



TexMATYC donated $370.00 to AMATYC New Orleans Fund.
Of this amount $119.44 was personal donations of three board members

Description	Income	Expenses
Balance (as off 2/21/07)	$5,265.35	
Membership	$335.00	
Conference Proceeds	$2,457.50	
Plaques and Mailing Expenses		$161.54
Interest	$18.84	
Donation to New Orleans Fund		$250.56
AMATYC Membership for award winners		$150.00
Total	$8,076.69	$562.10
Balance		$7,514.59

AMATYC Southwest Regional Conference Report

Heather Gamber

It was a pleasure to participate in the third AMATYC Southwest Regional Conference held in San Antonio on June 15 and 16, 2007. The last conference was held in 1997 in Flagstaff Arizona.

The conference was planned over the last two and a half years by a collaboration of immediate past presidents of OkMATYC, ArizMATYC, NMMATYC and TexMATYC. Dr. Linda Zientek represented Texas at the conference. Zientek, with the assistance of the TexMATYC board members, Sharon Sledge and Gerald Busald, was responsible for the local arrangements including hotel accommodations, continental breakfasts, lunches and snacks, shuttles, facilities such as room equipment and the conference bags. San Antonio College Mathematics Department, chaired by Dr. Said Fariabi, graciously made their facilities, equipment and computer labs available for the conference.

Many attendees arrived at the conference on Friday morning a little tired since they were kept awake Thursday evening by the noise of celebrations in the streets outside the hotel after the Spurs took their fourth NBA title.

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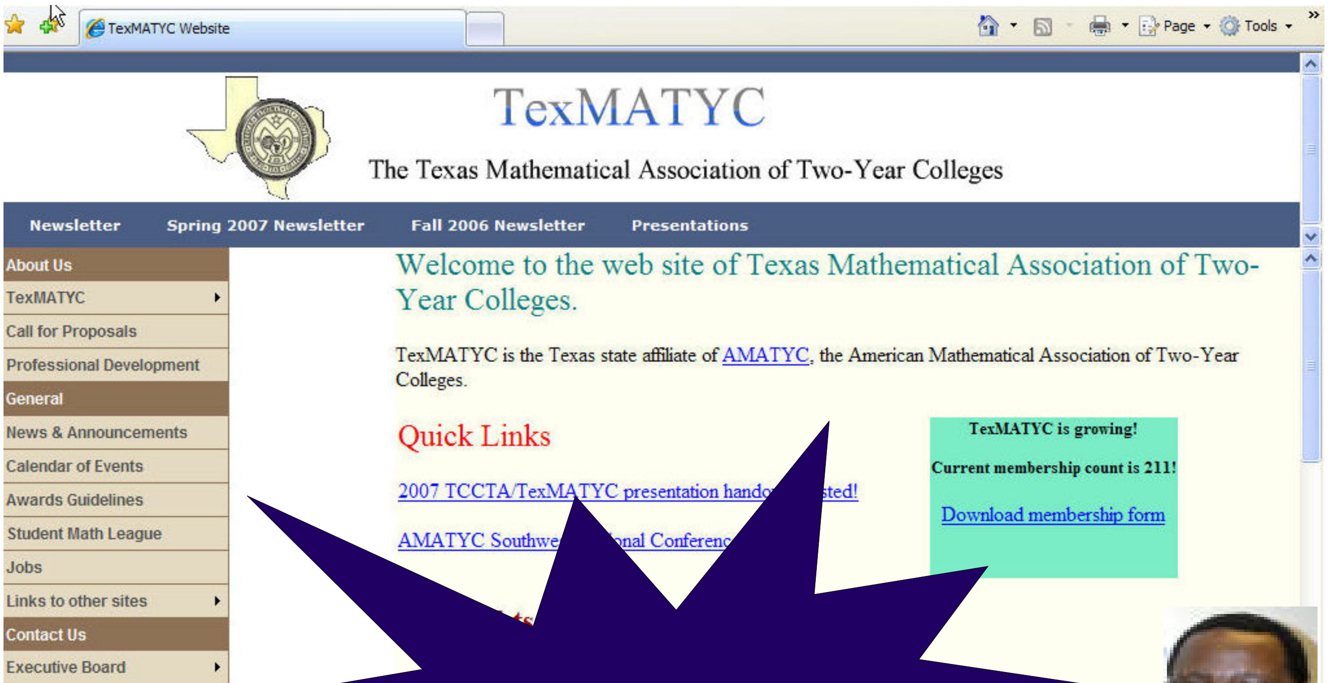
The conference opened with greetings from Dr. Robert Ziegler, SAC president, who spoke of his commitment to producing students who will be successful when they leave community colleges. Mary Robinson, AMATYC's Southwest regional vice president, welcomed us and introduced keynote speaker Dr. Joseph Gallian, MAA president and distinguished professor, who delighted us with a speech on rotations of motifs. AMATYC President Kathy Mowers introduced the lunch speaker Dr. Gloria White, who spoke on statewide data for P-16 mathematics education in Texas.

Over the two days, participants chose from approximately 50 talks, visited with the various textbook and calculator venders and visited with their colleagues from around the region. Participants left with new knowledge and valuable networking, and expressed a desire for this conference to be held more frequently.

Many of the presentations are available online at
www.swregion.matyc.org.

Dr. Gallion

Abigaill Baumgardner, Kisten Stoley



www.TexMATYC.org

Updated & Redesigned

Thank you, Charles Odion!

@ Student Email .edu

- Is CourseCompass correct on the due date for homework? I just wanted to make sure I wasn't doing all this homework too early.
- How do I send you an email?
- I couldn't take the test today because I couldn't find the College. My mom and I drove around _______ (*another county*) for two hours looking for it.

Refresher Courses in Introductory Algebra and Intermediate Algebra

Peg Crider

Tomball College and Collin County Community College have collaborated on developing curriculum for refresher courses in Introductory Algebra and Intermediate Algebra, and six one credit hour sequence of courses equivalent to Introductory/Intermediate sequence.

The refresher Introductory Algebra course targets students whose assessment is borderline Intermediate Algebra and the refresher Intermediate Algebra course targets students whose assessment is borderline College Algebra. These courses can be paired with second start courses. The targeted population of students may also include students who have credit in the course with a gap in time since last taking the course.

The modular courses is designed for repeater students, who may be able to demonstrate mastery of some of the learning objectives of a course. Rather than taking a full three-credit hour class, the students may take the course in one-hour components.

Both models are hybrid, relying on a computerized homework/ assessment package to supplement the curriculum. Also, both models are based on Accelerated Learning methodologies.

A Mathematics Applications Matrix that correlates application problems in career and technical education pathways identified by the Achieve Texas Project to the learning objectives in Introductory and Intermediate Algebra will also be available.

Finally, a survey of some Texas community colleges developmental mathematics learning objectives and assessment criteria will also be included in the resources.

The curriculum is undergoing editing, and is expected to be posted by mid-October on the website, http://accell-tx.nhmccd.edu.

Dates to Remember!

AMATYC Annual Conference
November 1-4, 2007
Minneapolis, MN

Nominations for
TexMATYC Teaching Excellence Award
December 15, 2007

TCCTA/TexMATYC Conference
February 22-23, 2008
Dallas, TX

ICTCM Annual Conference
Mar 6-9, 2008
San Antonio, TX

EMPLOYMENT

Cy-Fair College has an opening for a full-time college-level math instructor
www.nhmccd.edu
Employment
position 80496

Got News?

If you know of any exciting news in mathematics, have it published in your TexMATYC newsletter.
Submit articles to
Heather Gamber:
Heather.a.gamber2@nhmccd.edu

The Parabola as an Ellipse with Focus at Infinity

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Johannes Kepler claimed that a parabola is simply an ellipse with a focus at infinity [1, 2]–a description I first heard from my high school algebra teacher. Taken at face-value, this statement does not make sense. How can we say that a focus is 'at' infinity? Isn't infinity undefined? Below, one definition of infinity is given and it is shown that with this definition Kepler's statement is true. On the other hand, we do not have to work quite so hard to support the claim that 'a parabola is the limit of a family of ellipses where one focus *tends* to infinity' .

We begin by presenting a definition of conics that is particularly amenable to this study. Let l be a fixed line in the plane and F a point not on l. If X is a point in the plane, let d_1 be the distance from X to l, and let d_2 be the distance from X to F (see Figure 1). If e is a positive constant, the set

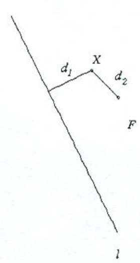

Figure 1: Illustration for Definition of Conic Sections

of all points such that $d_2/d_1 = e$ is a *conic section.* In this case, the line l is called the *directrix* and the point F is called a *focus.* From this definition it is not difficult to show that equations of the form

$$r = \frac{pe}{1 - e\cos(\theta)}, \tag{1}$$

are conic sections, where e is the eccentricity and p is the distance from a focus to the directrix [3]. Graphs of such equations with eccentricity 0.5, 1 and 2 are shown in Figure 2. The conic is an ellipse, parabola, or a hyperbola

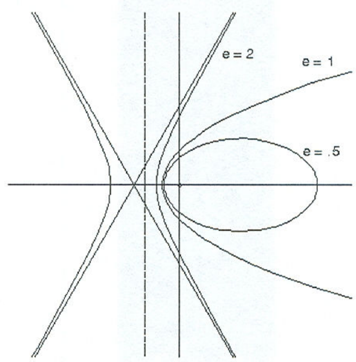

Figure 2: Conic Sections

depending on whether $0 < e < 1$, $e = 1$, or $e > 1$.

The formula on the right-hand-side of (1) is a continuous function of θ and e when $0 < e < 1$. For fixed θ not equal to 0, the limit of (1) as $e \to 1^-$ is then a point on a parabola. Also, one focus of ellipses of the form (1) is the origin and the other is at $(r = 2pe^2/(1 - e^2), \theta = 0)$, and so as $e \to 1^-$, it follows that one focus has a radius that tends to infinity. From this we see that the parabola is the pointwise limit of a family of ellipses with a focus that *tends* to infinity. However, this leaves open the question as to how a parabola *is* an ellipse with a focus *at* infinity. It is this question that will now be addressed.

Suppose a sphere S is tangent to the plane P, as illustrated in Figure 3. We identify points on the sphere with points on the plane as follows. The

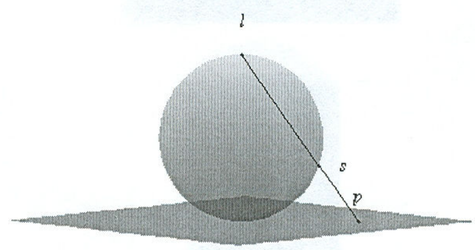

Figure 3: One Point Compactification; Stereographic Projection

line l through the pole of the sphere to a point p in the plane intersects the sphere at a point s. In this way, each point s on the sphere corresponds to one point p in the plane, except for the north pole. We may think of the sphere as being a representation of the plane with one point added, the north pole, which has an interesting interpretation. The farther the point p is from the origin, the higher the point s is on the sphere. Moving the point p out from the origin in any direction, the corresponding point s tends to the north pole. It is reasonable then to define the north pole of the sphere to be the *point at infinity.* This point, which we denote by ∞, is well-defined, and this construction gives the one-point compactification of the plane.

The projection of points on the sphere onto the plane, the exact opposite of this method, is called 'stereographic projection'. Ptolemy of Alexandria (late first and second century a.d.) described geometric properties of this projection in his treatise *Planisphaerium* where he suggests its use in map making [1]. If the plane is the complex plane, the sphere is called the *Riemann Sphere.* The addition of ∞ to the complex numbers allows one to consider the arithmetic of infinity–a topic we will not go into here. The topology of the one-point compactification was discovered by the Russian mathematician Aleksandrov [4, 5].

Now that we have a well-defined point at infinity, ∞, it may be possible to make sense of Kepler's statement. Suppose that the sphere has radius 1 and sits tangent to the xy-plane. Just as for terrestrial navigation where position on the earth is determined by latitude and longitude, every point on the sphere is determined by two angles, (θ, ϕ). The angle θ corresponds to longitude, though the angle ϕ is a bit different from latitude. It represents the angle from the north pole, to the center of the sphere S and then to the point s (see Figure 4).

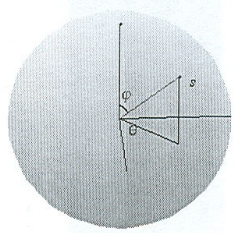

Figure 4: Coordinates on the Sphere

Figure 5 illustrates the relationship between polar coordinates on the plane and the coordinate ϕ on the sphere. The north pole, south pole and p form a right triangle, where the angle at the pole is $(\pi - \phi)/2$, and hence

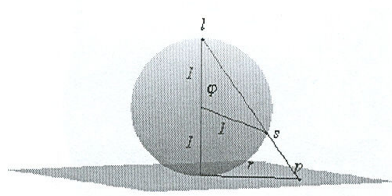

Figure 5:

$\tan[(\pi-\phi)/2] = r/2$. Therefore, we may translate between polar coordinates (r,θ) in the plane and coordinates (θ,ϕ) in the sphere by the relations $\theta=\theta$, and

$$r = 2\tan\left(\frac{\pi-\phi}{2}\right).$$

Conic sections in P given by (1) translate to curves in S given by

$$\phi = \pi - 2\tan^{-1}\left(\frac{pe/2}{1-e\cos\theta}\right) \tag{2}$$

as long as $1-e\cos\theta \neq 0$. When $1-e\cos\theta = 0$, we set $\phi = 0$, corresponding to ∞ on S. Conic sections with $e = 0.9$, $e = 1$, or $e = 2$ as they appear on the plane and the sphere are depicted in Figure 6.

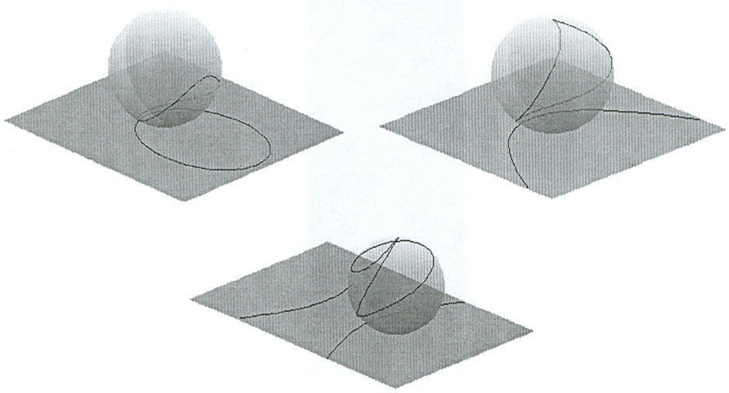

Figure 6: Ellipse, Parabola and Hyperbola

The location of one focus of the ellipse given in (1) is the origin and the other is located at $\theta = 0$ and

$$r = \frac{2pe^2}{1-e^2}. \tag{3}$$

On the sphere, this translates to $\theta = 0$, and

$$\phi = \pi - 2\tan^{-1}\left(\frac{pe^2}{1-e^2}\right). \tag{4}$$

Taking the limit as $e \to 1^-$, we have $\phi \to 0$, showing that on the one-point compactification of the plane a parabola is the limit of a family of ellipses with one focus tending to the well-defined point at infinity, ∞, on S. If we define the point $\theta = 0$ with ϕ as given in (4) to be a focus of the conic sections (2), then Kepler's statement that a parabola is an ellipse with a focus at infinity uses no undefined terms and is true. Indeed, it is not hard to see why it is sometimes said that hyperbolas are ellipses where one focus has been sent to infinity and back again from the 'other side', since on the sphere this is literally true. An animation illustrating this can be found at jgroah.nhmccd.cc/plot.gif, and at the National Curve Bank, curvebank.calstatela.edu/groah76/groah76.htm.

Kepler, in a treatise on optics published in 1604, describes a continuous transformation from conic to conic starting with two intersecting lines with 'foci' coinciding at the point of intersection. As the foci are separated, a family of hyperbolas is generated. When one focus is at infinity, only a parabola remains. As the focus moves beyond infinity, it approaches from the other side, and we pass from ellipse to ellipse until the two foci coincide again, yielding a circle [1].

Conclusion In the plane, the point at infinity is not defined and so in this context Kepler's statement that a parabola is an ellipse with a focus at infinity does not makes sense. However, on the one-point compactification of the plane Kepler's statement is valid.

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